

CSC&CSM – Mathematics A

T039

Tuesday, 20/11/2018

08:30 – 11:30 AM

WORKFORCE DEVELOPMENT AUTHORITY



P.O. BOX 2707 Kigali, Rwanda Tel: (+250) 255113365

**ADVANCED LEVEL NATIONAL EXAMINATIONS, 2018,
TECHNICAL AND PROFESSIONAL STUDIES**

EXAM TITLE: MATHEMATICS A

OPTIONS: Computer Science (CSC)

Computer Science and Management (CSM)

DURATION: 3 hours

INSTRUCTIONS:

The paper is composed of **the following Sections:**

Section I: Thirteen (13) compulsory questions. 55 marks

Section II: Attempt any three (3) out of five questions. 45 marks

All works must be done in the same booklet.

Silent, non-programmable scientific calculators may be used.

You may use a calculator and mathematical instruments.

Note:

Every candidate is required to carefully comply with the above instructions. Penalty measures will be applied on their strict consideration.

Section I. Thirteen (13) Compulsory questions**55 marks****01.** Solve

$$2^{4x+3} - 33(2^{2x-1}) + 1 = 0$$

5 marks**02.** Find the equation of the line through the given point perpendicular to the given equation: (7,2), $y = 4 - 5x$ **4 marks****03.** Consider $(u_n)_{n \in \mathbb{N}}$ as an AP such that $u_8 = 27$ and $u_2 + u_3 + u_4 = 21$. Calculate the common difference; write u_n as a function of n .**5 marks****04.** If $y = \sec x$, prove that $y \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2 + y^4$ **4 marks****05.** Three horses A, B and C are in race; A is twice as likely to win as B and B is twice as likely to win as C. What are their respective probabilities of winning, it means $P(A)$, $P(B)$ and $P(C)$?**4 marks****06.** Determine those values of t for which $\begin{vmatrix} t & 1 \\ 4t & t \end{vmatrix} = -3$ **3 marks****07.** Simplify the expression

$$\frac{\log_3\left(\frac{1}{3}\right)^{-3} - \log_3 243 + \log_3\left(27^{\frac{1}{2}}\right)^{\frac{8}{3}} + \log_3 a^3 + 1}{\log_3 a^2 + 2}$$

4 marks**08.** Prove that $\frac{\sin A + \sin 4A + \sin 7A}{\cos A + \cos 4A + \cos 7A} = \tan 4A$ **4 marks****09.** Show that $1 + i$ is a root of $z^4 - 4z^3 + 3z^2 + 2z - 6 = 0$. Hence find the other roots.**5 marks****10.** Find $\int x^2(4x^3 + 3)^4 dx$ **4 marks****11.** Prove by distances that the three points $(-2,3,5)$, $(1,2,3)$ and $(7,0,-1)$ are collinear.**5 marks****12.** The scores of 10 students in a test, in which the maximum marks were 50, were as follows: 28,36,34,26,48,24,35,27,21,41.

Find the variance and standard deviation.

4 marks

13. If the position vectors of the points A, B, C, D are $\vec{i} + \vec{j} + \vec{k}$, $2\vec{i} + 5\vec{j}$, $3\vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{i} - 2\vec{j} - \vec{k}$, then find the angle between the vectors \overrightarrow{AB} and \overrightarrow{CD} . **4 marks**

Section II. Choose and answer any three (3) questions

45 marks

14. a) Determine m in such way that the tangent on the curve $y = \frac{mx+1}{x+2}$ at the point $x=2$ is perpendicular to $y = -2x + 1$
b) Let f be the function $f(x) = \frac{2x^3+kx^2+2}{x^2-4}$ with k a parameter. Find k such that two asymptotes cut at A (2,4).
c. Find $\int x^2 \ln x dx$ **15 marks**
15. a. Determine the roots of the equation $z^3 + 64 = 0$ in the form of $a + ib$, where a and b are real.
b. If $\vec{a} = 2\vec{i} + 4\vec{j} - 3\vec{k}$ and $\vec{b} = \vec{i} + 3\vec{j} + 2\vec{k}$, determine:
(i) the scalar and vector products,
(ii) and the angle between the two given vectors. **15 marks**
16. a) Mr Kabera makes one set of chairs at a time each new set produced costs 8 000 FRW more than the previous one. Given that the first set costs 22 000 FRW, determine the total amount of money he received when 54 sets are sold.
b) Given that $4x^2 - 5x + 1 = 0$ has roots α and β .

Find the values of;

$$\alpha^3 + \beta^3$$

- c) Given α and $\alpha + 2$ are roots of $x^2 + 6x + q = 0$ and the roots of the equation

$$x^2 - qx + 8 = 0 \text{ are } \alpha \text{ and } \alpha - 2.$$

Find the value of q in the both cases.

15 marks

17. a) Write down the expression for the volume V and surface area, S of the cylinder of radius r and height h . If the surface area is kept constant, show that the volume of the cylinder will be maximum when $h = 2r$.

b. The values of the resistance of 90 carbon resistors were determined:

Resistance x ($M\Omega$)	2.35	2.36	2.37	2.38	2.9	2.40	2.41
Frequency (f)	3	10	19	20	18	13	7

Calculate: (i) the mode
(ii) the standard deviation

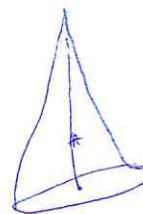
c. Solve the simultaneous equations:

$$\begin{cases} xy = 243 \\ 4(\log_x^x + \log_y^y) = 17 \end{cases} \quad \left| \begin{array}{l} 17 \\ 243 \end{array} \right.$$

15 marks

*18. Given the function $f(x) = \frac{x^2 + 2x + 1}{x + 3}$

- Find the domain of definition
- Study the parity
- Find the asymptotes to the curve
- Compute the first derivative
- Compute the second derivative
- Sketch the graph of $f(x)$.



15 marks

$$\begin{aligned} (x+3)^2 &= (x+3)(x+3) \\ &= x^2 + 3x + 3x + 9 \\ &= x^2 + 6x + 9 \end{aligned}$$

$$\begin{aligned} &2x - x)(x+3) \\ &2x^2 + 6x - x^2 - 3x \\ &x^2 - x^2 + 6x - 3x \\ &3 - 3x \end{aligned}$$